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Technical Report

Position of the Aircraft's Aerodynamic Center for $\frac{\partial c_M}{\partial \alpha} < 0$

For longitudional stability of an aircraft, an increased angle of attack needs to cause a compensating back torque:

$$\frac{\partial c_M}{\partial c_A}|_{CoM} \quad < \quad 0\,. \tag{0.1}$$

Equivlaent to Equation 0.1 is the requirement for the aerodynamic centre of the aircraft AC_A being located aft of the aircraft's centre of mass:

$$x_{AC} \quad > \quad x_{CoM}\,. \tag{0.2}$$

The aerodynamic centre of the Aircraft is defined as the point where changes in lift (in the lift coefficient c_A, respectively) do not cause changes in torque. Therefore, the location x_{AC} can be derived from

$$\frac{\partial c_M}{\partial c_A}|_{AC_A} \quad \overset{!}{=} \quad 0 \tag{0.3}$$

$$= \quad \frac{\partial c_{A_W}}{\partial \alpha} A_W x_{AC} - \frac{\partial c_{A_T}}{\partial \alpha}(l_W + l_H - x_{AC}) A_T\,, \tag{0.4}$$

where A_W and A_T are the areas of the main wing and the stabilizer, respectively. Resolved into the position of the aerodynamic centre Equation 0.4 becomes:

$$x_{AC} \quad = \quad \frac{\frac{\partial c_{A_T}}{\partial \alpha}(l_W + l_H)}{\frac{\partial c_{A_W}}{\partial \alpha}\frac{A_W}{A_T} + \frac{\partial c_{A_T}}{\partial \alpha}}\,. \tag{0.5}$$

In the following, we use the abbreviations $\frac{\partial c_{A_x}}{\partial \alpha} =: c_{Ax\alpha}$ and $\frac{1}{\frac{c_{AW\alpha}}{c_{AT\alpha}}\frac{A_W}{A_T}+1} =: k$. The demand for AC_A being located aft of CoM then translates into:

$$l_W \quad < \quad \frac{l_W + l_T}{\frac{c_{AW\alpha}}{c_{AT\alpha}}\frac{A_W}{A_T} + 1} \tag{0.6}$$

$$< \quad (l_W + l_T)k\,. \tag{0.7}$$

As both k and $(1-k)$ are positiv, it is:

$$l_W \quad < \quad \frac{k}{1-k} l_T\,. \tag{0.8}$$

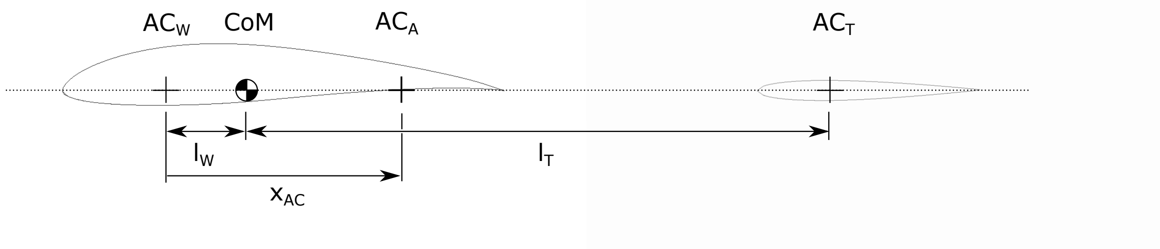

Abbildung 0.1: Geometry

Re-Substituting k leads to

$$l_W < \underbrace{\frac{A_T}{A_W}\frac{c_{AT\alpha}}{c_{AW\alpha}}}_{>0} l_T, \tag{0.9}$$

or:

$$l_T > \frac{A_W}{A_T}\frac{c_{AW\alpha}}{c_{AT\alpha}} l_W. \tag{0.10}$$

We assume the induced angle of attack at the stabilizer being proportional to the angle of attack of the main wing

$$\alpha_{T,ind} = s\alpha \tag{0.11}$$
$$\alpha_T = (1+s)\alpha, \tag{0.12}$$

where s is a constant value emerging from the mainwing's wake. Both ram pressure loss and effects due to 3-dimensional flow (in particular different aspect ratios of stabilizer and main wing) are neglected. Equation 0.10 then becomes:

$$l_T > \frac{A_W}{A_T}\frac{1}{1+s} l_W. \tag{0.13}$$

For a conventional aircraft configuration s will most likely be negative, as the stabilizer is affected by the mainwing's downwash. Also neglecting the influence of the induced angle of attack ($s = 0$) one gets the simple and visceral relation:

$$l_T A_T > l_W A_W. \tag{0.14}$$

Note that neither the lift coming from the mainwing nor the lift produced by the stabilizer shows up here. The demand for having a compensating back torque only depends on geometry and the ratio of $c_{AW\alpha}$ and $c_{AT\alpha}$. For $l_W < 0$ (AC_W being located aft of CoM) Inequation 0.14 is true without having any demands for the stabilizer's lever and area. Indeed, such an aircraft would be stable in terms of requirement 0.1 without even having a stabilizer. However, for a conventionally cambered mainwing no setpoint with positive lift would be found. The demand for the stabilizer's lever and area as in Equation 0.13 is not limited to conventional aircraft configurations but also for canard configurations. For the latter case one would either have to swap the values refered to main wing and stabilizer, or assume that at least l_T would be negative.