DC Motor Formulas V1.4

I_o...no load current [A]

$I_{o(Unew)} \approx I_o * (U_{(new)} / U_{(Io)})^{0.75}$

R_i...motor winding resistance [Ohm]

K_v...motor velocity constant [rpm/V]

K_t...motor torque constant [Nm/A]

U_{motor}...motor supply voltage, after ESC [V]

I_{motor}...motor current [A]

rpm…motor speed [revolutions per minute]

Formula for motor torque constant:

$K_t = (1 / K_v) * (30 / pi)$ [Nm/A]

Formulas for output power:

$rpm = (U_{motor} - R_i * I_{motor}) * K_v$

motor speed [rad/s] = rpm * pi / 30

P_{out} = motor speed * $(I_{motor} - I_o) * K_t$

or

$I_{stall} = U_{motor} / R_i$ [A]

$P_{out} = (I_{motor} - I_o) * (I_{stall} - I_{motor}) * R_i$

I_{motor} at $P_{out}(max) = I_o / 2 + I_{stall} / 2$

Formula for max efficiency:

$\eta_{max} = (1 - \sqrt{(I_o / I_{stall})})^2$

Formulas for efficiency:

$\eta = 1 + I_o / I_{stall} - I_{motor} / I_{stall} - I_o / I_{motor}$

$\eta_{(Pmax)} \approx ½ - I_o / I_{stall}$

$\eta_{(Istall/6)} = 0.83 - 5 * (I_o / I_{stall})$

Formula for $I_{motor}(\eta_{max})$:

$I_{motor}(\eta_{max}) = \sqrt{(I_{stall} * I_o)}$

Formula for I_{motor}:

$I_{motor} = (K_v * U_{motor} - rpm) / K_v / R_i$

Formula for $R_i(t)$:

$R_i(t) = R_i(t_o) (1 + \alpha_{to} * (t - t_o))$

$t_o = 20℃$

$\alpha_{to} = 3.9 * 10^{-3}$ [1/K]

$R_i(90℃) = R_i(20℃) * 1.27$

⇨ ***good motor cooling is recommended!***

Formula for loaded motor:

Now we use a prop as motor load.

The static power consumption of a prop can be described with the n_{100} constant.

n_{100}…propeller speed [rpm] where the input power of the propeller is 100 W

$P_{prop} = 100 W * (rpm / n_{100})^3$

To obtain the loaded motor speed, the following system of equations must be solved:

(1)…$(I_m - I_o) * (I_{stall} - I_m) * R_i – P_{prop} = 0$
(2)...$I_m = (K_v * U_{motor}) / K_v / R_i$
(3)...$P_{prop} = 100 W * (rpm / n_{100})^3$

Now also a gearbox is used.

The propeller speed is reduced by the gear ratio Z

The gearbox efficiency $\boldsymbol{\eta}_g$ is assumed as constant.

A good gearbox can be assumed with an efficieny of 0.96.

P_{prop} is now:

(31)…$P_{prop} = (100 W * ((rpm / Z) / n_{100})^3) / \boldsymbol{\eta}_g$

Result of solving the system of equations (1), (2), (31) leads to the following equation:

$$rpm= - 1 / 200 * Z^3 * n_{100}^3 * \boldsymbol{\eta}_g * (1 / K_v^2 / R_i - (Z^3 * n_{100}^3 * \boldsymbol{\eta}_g + 400 * K_v^3 * R_i * U_{motor} – 400 * K_v^3 * R_i^2 * I_o)^{(1/2)} / K_v^2 / R_i / Z^{(3/2)} / n_{100}^{(3/2)} / \boldsymbol{\eta}_g^{(1/2)})$$

Remark: Matlab calculation file see annex on last page

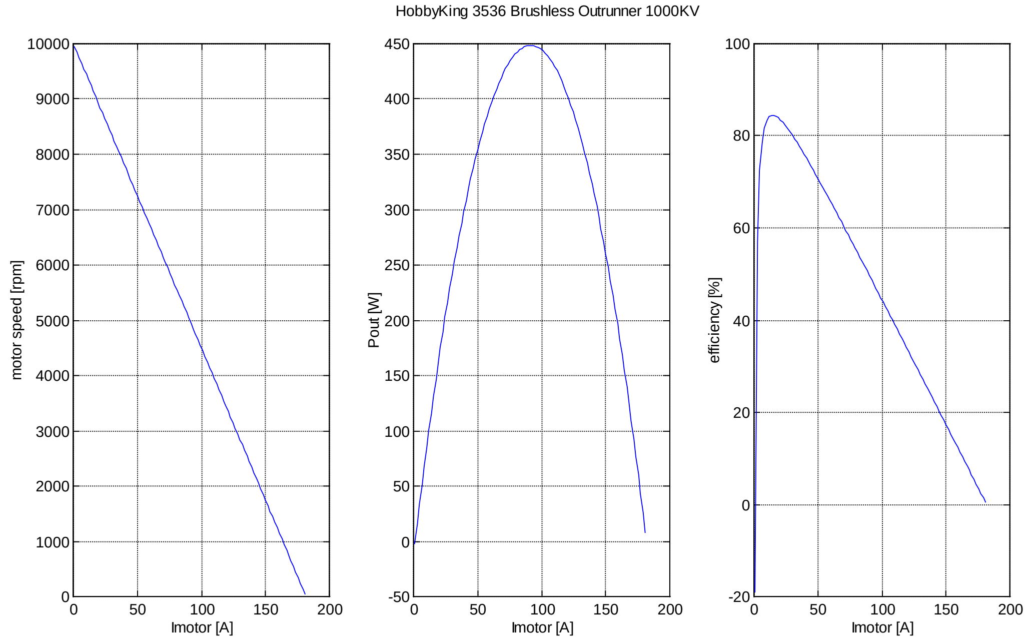
DC Motor graph:

Example: HobbyKing 3536 Brushless Outrunner 1000KV:

U_{motor} = 10.0 V
K_v = 1000 rpm/V
R_i = 0.055 Ohm
I_o = 1.1 A

$\eta_{max} = (1 - \sqrt{(I_o / I_{stall})})^2$
$= 0.85$

$I_{motor}(\eta_{max}) = \sqrt{(182A * 1.1A)}$
$= 14.15$ A



Motor graph of rpm, Pout and motor efficieny [%]

Annex

Matlab File (Version 4.2c.1) for loaded motor (Propeller):

```
% Equations DC Motor plus Prop
clc;
G1='(((Kv*Umotor-rpm)/Kv/Ri)-Io)*(Umotor/Ri-((Kv*Umotor-rpm)/Kv/Ri))*Ri-
(100*(rpm/Z/n100)^3)/etha_g=0';
%G2='((Kv*Umotor-rpm)/Kv/Ri)-Im=0';
%G3='(100*(rpm/Z/n100)^3)/etha_g -Pprop=0';

%[rpm] = solve(G1,'rpm')

%rpm= -1/200*Z^3*n100^3*etha_g*(1/Kv^2/Ri-(Z^3*n100^3*etha_g+400*Kv^3*Ri*Umotor-
      400*Kv^3*Ri^2*Io)^(1/2)/Kv^2/Ri/Z^(3/2)/n100^(3/2)/etha_g^(1/2))
```

Useful links

n_{100} values for aero-nout props:

http://www.aero-naut.de/produkte/flugmodellbau/flugzubehoer/luftschrauben/classicprop.html

see Downloads -> N-100 Werte

some more about Prop data:

http://www.ae.illinois.edu/m-selig/props/propDB.html#MAS

other useful site with motor and prop data:

http://www.flybrushless.com/

Recommendation for a reasonable Motor Input Power:

As rule of thumb, a motor input power of 3W / gram (Motor weight) should be not exceeded.

Motor input power $P_{in} = U_{motor} * I_{motor}$